EQUIVALENT COAXIAL TRANSMISSION LINES

Sheng-Gen Pan

Department of Radio Electronics
Shanghai University of Science and Technology
Shanghai 201800, China

Abstract

A family of transmission lines is based on a circular conductor and a noncircular conductor. Two new types of equivalent eccentric coaxial lines, which give smooth transition between extremes of a small wire and a wire near contact, are presented. The results obtained are very simple analytical expressions which will be useful for fast computation or for the CAD of coaxial components. The accuracy of the expressions is confirmed by comparing with accurate numerical data.

I. Introduction

The determination of the characteristic impedance of a coaxial system consisting of a circular conductor and a noncircular conductor has been the subject of numerous treatments appearing during the past forty years [1-10]. We may use three approachs to calculate its impedance 1) conformal transformation, 2) numerical techniques, and 3) graphically approximate methods which identify an equivalent coaxial transmission line whose impedance is well known and is expected to be similar to that of the one under investigation. The third method has been used extensively to produce an equivalent circular coaxial line at samll ratios of inner and outer conductors[2,3]. However, this approach does not take into account the interaction of inner and outer conductors. Recently an equivalent eccentrical coaxial line was presented for a coaxial line consisting of a noncircular outer conductor and a circular inner conductor[5]. In this paper, we further develope the approximate graphical method and presented two new types of equivalent eccentrical coaxial lines, whose eccentricities vary with the ratio of inner and outer conductors, for general coaxial systems. The new equivalent lines give smooth transition between extremes of a small wire and a wire near contact. The results obtained are very simple analytical expressions which will be useful for fast computation of the characteristic impedance or for the CAD of coaxial components.

II. Theory and Method

The interior of a unite circle in the φ-plane can be conformally mapped into the interior of a simple connected region in the w-plane by an infinite series of the form[6]

$$W=\sum_{n=0}^{\infty}\alpha_n \varphi^{1+n} \tag{1}$$

The exterior of a unite circle in the φ-plane can be conformally mapped onto the exterior of a simply connected region in the w-plane by an infinite series of the form [6]

$$W=\sum_{n=0}^{\infty}\beta \varphi^{1-n} \tag{2}$$

When the ratio $r/R \rightarrow 0$, the effective radius of a noncircular outer conductor is given from (1) with close approximation by

$$R_{eo}=|\alpha_0| R \tag{3}$$

and the effective radius of a noncircular inner conductor is given from (2) with close approximation by

$$r_{eo}=|\beta_0| r \tag{4}$$

The coefficients α_n and β_n can be systematically determined by numerical methods, such as successive approximations, Melentiev's method, etc[6]. The coefficient α_0 or β_0 also can be obtained by means of the closed analytic function.

Considering the basic properties of the effective radius given in [5], we propose to use eccentrical coaxial lines, whose eccentricities vary with the ratios of inner and outer conductors, as the equivalent coaxial lines. At the extreme of a small wire ($r/R \rightarrow 0$), the eccentricities of the equivalent lines are zero and the equivalent lines become circular coaxial lines with the effective radius R_{eo} in (3) for a noncircular outer conductor and with the effective radius r_{eo}

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in (4) for a noncircular inner conductor.

At the extreme of a large wire near contact ($r/R \rightarrow 1$), the inner and outer conductors of the equivalent lines should be near contact. For the case r/R=1, the eccentricity of the equivalent lines reaches the maximum value, and can be determined easily from the geometrical configuration of a eccentrical coaxial line. In particular, the maximum eccentricities of the equivalent lines are

$$E_{max} = \left(1 - \frac{1}{|\alpha_0|}\right) \tag{5}$$

for a coaxial system consisting of a noncircular outer conductor and a circular inner conductor, and

$$E_{max} = 1 - |\beta_0| \tag{6}$$

for a coaxial system consisting of a circular outer conductor and a noncircular inner conductor.

Between the extremes of a small wire and one near contact, we may choose the eccentricity of an equivalent eccentric line such that its effective radius is very close to that of the one under investigation. We use $|\alpha_0|$ and $|\beta_0|$ to indicate the departure of a noncircular conductor from a circular one. Thus, the eccentricities of the equivalent line should be a function of $|\alpha_0|$, $|\beta_0|$ and r/R.

Although it is difficult to solve for the effective radius of a coaxial conductor of any irregular cross section at any ratio r/R, there are some combinations of circular and noncircular conductors whose effective radii can be evaluated exactly. These can be used to determine the function of the eccentricity of the equivalent line with the variables $|\alpha_0|$, $|\beta_0|$ and r/R by means of optimization techniques, such as optimum seeking methods.

Once the such a function is determined, the characteristic impedance at any ratio r/R of a coaxial system consisting of circular and noncircular conductors can be easily calculated by the formula for the determination of the characteristic impedance of the eccentrical coaxial line.

III. Examples

1. Noncircular outer conductor

The eccentricity of the equivalent eccentrical line for a coaxial system consisting of a noncircular outer conductor and a circular inner conductor is chosen as

$$E_1(r/R) = \left(1 - \frac{1}{|\alpha_0|}\right)(r/R)^{F_1(r/R)} \tag{7}$$

where

$$F_1(r/R) = \frac{2}{|\alpha_0|}\left[1 - (r/R)^{\frac{10}{|\alpha_0|}}\right] \tag{8}$$

Then, the formula for the determination of the characteristic impedance of this kind of coaxial transmission line is given by

$$Z_0 = 59.952 \ln\left[G + \sqrt{G^2 - 1}\right] \tag{9}$$

where

$$G = \frac{1}{2}\left\{\frac{2r}{|\alpha_0|R} + \frac{|\alpha_0|R}{r}\left[1 - E_1(r/R)\right]\left[1 - \frac{(1 - E_1(r/R))}{2}\right]\right\} \tag{10}$$

To show the validity of the formula given above, we consider some typical examples. Fig. 1 shows a family of outer conductor cross sections that are regular polygons. α_0 for polygons can be formulated in terms of Gamma function[3]

$$\alpha_0 = \Gamma(1 + \tfrac{2}{n}) / \Gamma^2(1 + \tfrac{1}{n}) \tag{11}$$

Substituting (11) into (7-10) yields simple analytical expressions for the characteristic impedance of a coaxial system consisting of an N-regular outer conductor and a circular inner conductor. Table 1 shows a comparision between the characteristic impedance reported in the literature and obtained using these simple expressions. It is shown by Table 1 that the agreement is excellent in all case. In particular, the maximum deviation is less 0.2 percent in the range r/R<0.5 and less 0.7 percent in the rang r/R<0.95. At the extreme of large wires near contact, we find, by taking the limit of (9), that results obtained here are very close to Wheeler's limiting results for polygons[4]

2. Noncircular inner conductor

The eccentricity of the equivalent eccentrical line for a coaxial system consisting of a circular outer conductor and a noncircular inner conductor is chosen as

$$E_2(r/R) = (1 - |\beta_0|)\left(0.66 + \frac{|\beta_0|}{10}\right)^{g(r/R)}(r/R)^{F_2(r/R)} \tag{12}$$

where

$$g(r/R) = 1 - (r/R)^{80} \tag{13a}$$

$$F_2(r/R) = \left(3\sqrt{|\beta_0|}\right)^{\left[1 - (r/R)^{15}\right]} \tag{13b}$$

Then, the formula for the determination of the characteristic impedance of this kind of coaxial transmission lines is given by

$$Z_0 = 59.952 \ln\left[G + \sqrt{G^2 - 1}\right] \tag{14}$$

where

$$G = \frac{1}{2}\left\{\frac{2|\beta_0|r}{R} + \frac{R}{|\beta_0|r}\left[1 - E_2(r/R)\right]\left[1 - \frac{(1 - E_2(r/R))}{2}\right]\right\} \tag{15}$$

Fig. 2 shows a family of inner conductor cross sections that are regular polygons. β_0 for polygons can be formulated in terms of Gamma function[3]

$$\beta_0 = \frac{n\Gamma^2(1 + 1/n)\sin(\pi/n)}{\pi\Gamma(1 + 2/n)} \tag{16}$$

Substituting (16) into (12-15) yields simple analytical expressions for the characteristic impedance of a coaxial system consisting of a circular outer conductor and an N-regular polygon inner conductor. Table 2 shows a comparision between the characteristic impedance reported in the literature and obtained using these simple expressions. It is shown by Table 2 that the agreement is excellent in most cases. In particular, the maximum deviation is less 0.3 percent in the range $r/R<0.9$ for N=2, 4 and 6. For N=3, our results are almost 1.5Ω greater than those of Sheshadri's for different r/R ratios. By the theory of small wire theory, the results reported in this paper are found to be more accurate than those reported by Sheshadri[9].

VI. Conclusion

The new-type equivalent eccentrical coaxial lines have been presented for general coaxial systems. The elementary foumulas for characteristic impedance are given in a form that can be implemented easily on a pocket calculator. In terms of their accurary, the described formulas represent a considerable improvement to the foundation of coaxial component design and are fully compatible with the needs of modern computer-aided microwave coaxial circuit design.

References

[1] M. A. Gunston, Microwave Transmission Line Impedance Data, New York: Van Nostrand Reinhold, 1972.
[2] S. Frankel, Multiconductor Transmission Line Analysis, Dedlam, NA: Artech-House, 1978.
[3] H. A. Wheeler, IEEE Trans. Microwave Theory Tech. vol.MTT-28, pp.73-83, Jan. 1980.
[4] H. A. Wheeler, IEEE Trans. Microwave Theory Tech., vol.MTT-27, pp.712-21, Aug. 1979.
[5] S. G. Pan, IEEE Trans. Microwave Theory Tech., Vol.MTT-35, pp.61-63, Jan. 1987.
[6] L. V. Kantorovitch and V. Krylov, Approximate Methods of Higher Analysis, New Youk; Interscience Publishers, Inc., 1958.
[7] H. J. Riblet, IEEE Microwave Theory Tech., vol.MTT-31, pp.841-44, Oct. 1983.
[8] T. K. Sheshadri and K. Rajaian, Proc. IEEE, vol.70, pp.82-83,Jan. 1982.
[9] T. K. Sheshadri and K. Rajaian, IEE Proc. H, vol.131, pp.279-80, Aug. 1984.
[10] L. N. Epele, H. Fanchiotti, C. A. G. Canal and H. Vucetich, Proc. IEEE, vol.72, pp.223-24, Feb. 1984.

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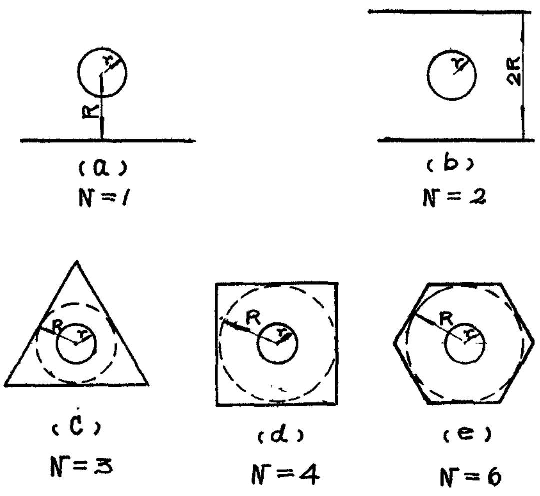

Fig. 1 Outer conductor of regular-polygon cross section

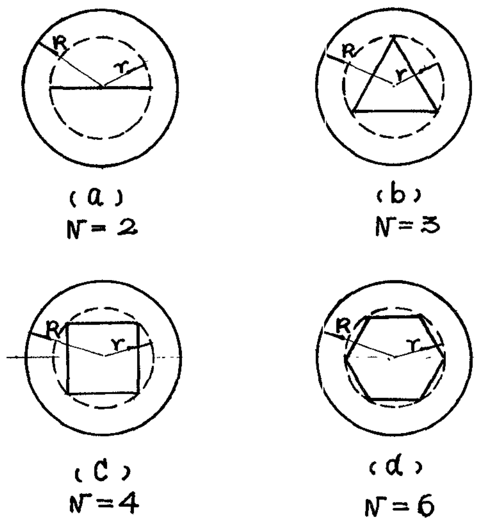

Fig. 2 Inner conductor of regular-polygon cross section

Table 1 The characteristic impedance for N-regular polygon outer conductor

N	1		2		6	
r/R	present work	Gunston [1, p.16]	present work	Wheeler [4]	present work	Seshagiri [1, p.89]
0.05	221.12	221.12	194.08	194.08	181.82	181.64
0.1	179.45	179.45	152.52	152.52	140.26	140.01
0.3	112.34	112.34	86.59	86.62	74.40	74.04
0.5	78.95	78.95	56.66	55.72	43.77	43.43
0.7	53.69	53.69	34.51	34.54	23.56	23.34
0.9	28.01	28.01	16.03	16.07	8.30	8.25
0.95	19.37	19.37	10.64	10.67	4.85	4.83
N	3			4		
r/R	present work	Seshadri [8]	Epele [10]	present work	Seshadri [8]	Riblet [7]
0.05	187.04	187.32		184.14	184.42	
0.1	145.48	145.70	145.50	142.59	142.50	
0.3	79.61	79.74	79.63	76.72	76.84	
0.5	48.91	49.03	48.98	46.07	46.16	46.09
0.7	28.43	28.57	28.53	25.77	25.89	25.85
0.9	11.99	12.06	12.10	10.06	10.15	10.13
0.95	7.70			6.24		6.25

Table 2 The characteristic impedance for N-regular polygon inner conductor

N	3		4	
rsin R	present work	Sheshadri [9]	present work	Sheshadri [9]
0.05	156.87	155.20	169.66	169.59
0.1	115.31	113.58	128.11	127.95
0.3	49.24	47.40	62.23	61.99
0.4	31.01	28.86	44.94	44.70
0.5			31.40	31.20
0.6			19.87	19.76
0.65			14.24	14.08
N	2		6	
rsin R	present work	Gunston [1, p.80]	present work	Sheshadri [9]
0.05	221.16	221.15	175.95	
0.1	179.60	179.60	134.39	
0.3	113.69	113.67	68.54	68.59
0.6	71.22	71.76	26.93	26.96
0.7	60.99	60.95	17.58	
0.9	40.80	40.93		
0.94	36.21	36.00		